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For Niklas Luhmann:* “How Recursive is Communication?”

HEINZ VON FOERSTER

(Translated by Richard Howe)

A year and a half ago Niklas Luhmann sent me a fascinating essay for my 80th birthday (Luhmann 1991). This article culminates in two extraordinary questions. I won't read you these questions now;[§] I'd rather just briefly report on the impression these questions made on me. I saw in them a resemblance to two of the great problems of antiquity, two problems of geometry. The one problem is the *Trisection anguli*. That is the problem of dividing an angle into three parts using only a compass and a ruler. And the other problem is the *Quadratura circuli*, the task of constructing a square, again using only a compass and a ruler, the area of which is equal to a given circle. As you probably recall, both of these problems are unsolvable in principle, as Karl-Friedrich Gauss showed about two hundred years ago. But if one removes the restriction of working only with a compass and a ruler, then these problems can easily be solved.

When I got the invitation to say a few words here at the birthday celebration for Niklas Luhmann, I of course immediately thought, oh good, now that's where I'll present my answers to the two problems that he put to me for my birthday. I sat myself down and went to work on the answers, but in the midst of my preparations it suddenly occurred to me: but Heinz, that's

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[§] Editor's note: The two questions run (Luhmann 1991, p. 71) “1. Does knowledge rest on construction in the sense that it only functions *because* the knowing system is operatively closed, therefore: *because* it can maintain no operative contact with the outside world; and because it *therefore* remains dependent, for everything that it constructs, on its own distinction between self-reference and allo-reference? 2. Can (or must) one impute the formation of “Eigen values” to the domain of latency; therefore for first order observation to the intangible and therefore stable distinction that underlies every single designation of objects; and in the domain of second order observation to those very forms that are conserved when a system interrupts its constant observation of that which cannot be observed?”

completely wrong! One just doesn't do that in this part of the world. Here one gives birthday children questions, not answers! So therefore I thought to myself, okay, I'll present my answers at some other opportunity; today, on the occasion of this birthday celebration, I too will come with two questions. And it's not just about questions but rather—we are, indeed, here in the Center for Interdisciplinary Research—about two research programs into still unsolved problems of the social sciences. I thought I'd present these problems today, for I have the feeling that if one would concern oneself with these questions one could make an essential contribution to social theory.

What are these two questions about? The first problem or research program has to do with an extension, or perhaps I should say: with a deepening, of recursive functions. You all know about the unprecedented successes of the recursive functions that are in constant use in chaos theory and indeed elsewhere. But I have the feeling that these results of chaos research can be applied by sociology only metaphorically. Why? All chaos research is concerned with functions, and functions are only relations between numbers, at best, complex numbers. A function can be quadratic, one gives this function a two, out comes a four, and one gives this function a three, and out comes a nine. It operates only on numbers, but sociology doesn't work with numbers: sociology is interested in functions. And functions of functions one calls functors. A functor is, so to speak a system that is intended to coordinate one group of functions with another group, and so today I propose to develop a research program in which one is concerned with recursive functors. So that's problem number one.

Problem number two that I'd like to present today is a theory of compositions. It consists in developing a system of composition, and, indeed, a system of composition for systems. What is this problem about? I have System **A**, I have a System **B**, and now I'd like to integrate both of these into a System **C**. What do the rules consist of that allow a new System **C** to arise, the rules of integration, of composition? Is it a kind of addition, a kind of integration? We've got all the best words for it, but what does the formalism for such problems look like? Today one could also provide the composition problem with another name: It's about, for example, the problem of the Croats, the Bosnians, the Herzegovinians—one could call it the Vance-Owen Problem. These are the problems that we confront in social theory today. How can one solve this problem? Or in a different sense it is also about the problem of autopoiesis: how can I bring an autopoietic System **A** into a relationship with another autopoietic System **B** in such a way that a new System **C** arises, itself an autopoietic system? Unfortunately, the poets or autopoets who invented autopoiesis have given us no rules for the compositional possibilities of such autopoietic systems. They have, to be sure, applied indices, but that isn't really a fundamental theory of composition. These are, in brief, my two problems.

Now of course you'll say, "for heaven's sakes, we're sociologists, and here Heinz von Foerster comes with fundamental mathematical problems—what

are we supposed to make of that?” So I thought I could sweeten or lighten this problematique if I tried above all to present the ideas so clearly that they became transparent. And when something is transparent, then one no longer sees it: the problems disappear. And as a second idea, I thought I’d bring from California a trio of American jewels for our birthday child Niklas Luhmann that are probably somewhat known here already but may still amuse the birthday child in their special birthday edition.

The first present I’ve brought along is an essay by Warren McCulloch, written about a half a century ago. It is the famous article with the title “A Hierarchy of Values Determined by the Topology of Nervous Nets” (1945). I find that this article is of such great significance that I’d like to draw your attention to its existence once more. So that you can see what field he worked in, I’ll read you a sentence from the last paragraph. It rests on the idea of a circular organization of the nervous system: “circularities in preference.” These circularities arise when one prefers **A** to **B**, prefers **B** to **C**, and, again, **C** to **A**. In classical logic one then speaks of being illogical. Nevertheless, McCulloch says that is not illogical, it is logic as it is actually used. Therefore: “Circularities in preference instead of inconsistencies, actually demonstrate consistency of a higher order than had been dreamed of in our philosophy. An organism possessed of this nervous system—six neurons—is sufficiently endowed to be unpredictable from any theory founded on a scale of values.” A system of six neurons is, in the framework of existing theories, unpredictable in principle. That is present number one.

Present number two that I’ve brought along with me is an article by Louis Kaufmann, a mathematician who is fascinated by self-reference and recursion. The article is called “Self-reference and recursive form” (1987). And so that you can see why I find it so important, I’ll read you the last sentence of this article. The last sentence of this article is: “Mathematics is the consequence of what there would be if there could be anything at all.”

Present number three is by my much admired teacher Karl Menger, a member of the *Wiener Kreis* (Vienna Circle), to which I am pleased, even today, to have fallen victim! When I was a young student I enthusiastically attended Karl Menger’s lectures. The article by Karl Menger, which I’ve brought here as present number three, is “Gulliver in the Land without One, Two, Three” (1959). You may ask, why I’ve brought such an article to a group of sociologists! In this article Karl Menger already developed the idea of functors, that is, of functions of functions, which I consider wholly decisive for the theoretical comprehension of social structures. Here too I’ll read the last sentence, so you can see what it’s all about. The last sentence is: “Gulliver intended to describe his experiences in the Land without One, Two, Three in letters to Newton, to the successors of Descartes, to Leibniz, and to the Bernoullis. One of these great minds, rushing from one discovery to the next, might have paused for a minute’s reflection upon the way their own epochal ideas were expressed. It is a pity that, because of Gulliver’s preparations for another voyage, those letters were never written.”

So now I'd like to turn these three presents over to Niklas Luhmann! Of course that requires a bouquet!"*

Now I come to a theme that is not my own but rather was proposed to me by the Center for Interdisciplinary Research. I always like it when one proposes a theme for me, for if I then come with this theme, I optimally fulfill the wishes of my hosts! The assignment that was posed for me for today consists of the question "How recursive is communication?" I didn't know how I was supposed to read that. *How* recursive is communication? Or: how *recursive* is communication? Or, how recursive *is* communication? Unfortunately I'm not an ontologist, i.e., I don't know what *is*. I've never been able to do anything but consider what would be—if. So I've posed the question to myself as how would it be *if* we conceived of communication as recursion. And so here is my Proposal No. 00.

00. Proposal: "Communication is recursion."

You could understand that as if it had to do with an entry in a dictionary. If you don't know what communication is, you look it up in the dictionary under *C*. There it says, "Communication is recursion." Aha, you say, good! What is recursion? Then of course you go back to the dictionary again and find, this time under *R*: "Recursion is communication." So it is with every dictionary. If you busy yourself a bit with them, you will find that the dictionary is always self-referential: From **A** you are sent to **B**, from **B** to **C**, and from **C** back again to **A**. That's the dictionary game. You could of course also conceive of my proposal as a simple tautology: "communication is recursion." Indeed, but, as the philosophers assert, tautologies don't say anything. Nevertheless, tautologies do say something about the one who utters them. At the end of my lecture you may not know anything about recursion or communication, but you will certainly know something about me! My program, therefore, is the proposal: "communication is recursion," and what it looks like.

I'd like to present my program in three chapters, whereby I'd like to use the first chapter essentially to recall to your memories a terminology whose central concept is a fictitious "machine" that executes well defined operations on numbers, expressions, operations, etc. This chapter starts out by recapitulating some concepts that are already current among you. As you will later see, I'm using this terminology in order to make the decisive point in my lecture palatable to you, namely, insight into the unsolvability, in principle, of the "analytical problem." In other sciences this problem goes under other names: it's called "the decision problem" in logic, the "halting problem" in computer science, etc.

* Editor's note: Heinz von Foerster hands Niklas Luhmann copies of the three articles, specially bound for this occasion; as he does so, a bouquet of flowers appears magically out of thin air and Niklas Luhmann thanks him.

I gave a lot of thought to what version of this problem I could acquaint you with so that, without resorting to mathematical somersaults, the abyss dividing the synthetic problem from the analytic one would become clear. I finally allowed myself a compromise, in that I won't demonstrate how the analytical problem is unsolvable *in principle* but rather only an easier version, namely, that all the taxes in the world and all the time available in our universe would by no means be sufficient to solve the analytical problem for even relatively simple “non-trivial machines”: the problem is “transcomputational,” our ignorance is fundamental.

This abysmal ignorance, this complete, fundamental ignorance is something that I've still never really seen presented at full strength, and that is what I'd like to present to you today, so that we can get some insight into the question of how, in the face of such fundamental ignorance, we can concern ourselves with our problems? In the second chapter then I'd like to sketch the development of recursive functors. I'll make it as easy and playful as possible, so you can enjoy following these trains of thought. And in the third chapter I'd like to speak about compositions, compositions of functors, of compositions of systems.

First Chapter: Machines

I'll begin the first chapter by recapitulating a language that was actually introduced by Alan Turing, an English mathematician, in order to leave the long-windedness of deductively logical, argument to a machine, a conceptual machine, that would then turn all the wheels and buttons, so that one only has to observe it: if one enters the problem on one side of the machine, then the solution emerges on the other side. Once this machine has been established, we have a language that can very easily jump from one well-defined expression to another, and if you then want to know how this machine works, you can always take it apart. Therefore: machine language. This language is already current among you, but permit me, despite that, to briefly repeat it, for, as I said, I'll need to concepts in a few minutes.

I come to my proposition:

01. Trivial machines: (i) synthetically determined; (ii) independent of the past; (iii) analytically determinable; (iv) predictable.

A trivial machine is defined by the fact that it always bravely does the very same thing that it originally did. If for example the machine says it adds 2 to every number you give it, then if you give it a 5, out comes a 7, if you give it a 10, out comes a 12, and if you put this machine on the shelf for a million years, come back, and give it a 5, out will come a 7, give it a 9, out will come an 11. That's what's so nice about a trivial machine.

But you don't have to input numbers. You could also input other forms. For example, the medieval logicians input logical propositions. The classi-

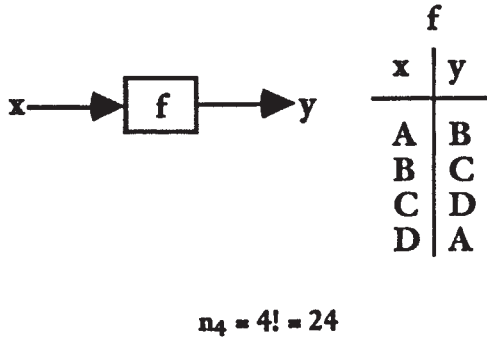


FIGURE 1.

cal logico-deductive proposition that was always used as an example in the Middle Ages is the famous proposition “All men are mortal.” So you arrive at an “All men are mortal”-trivial machine. If you shove a person into it on one side, a corpse comes out the other. Take Socrates—“Socrates is a man”—, shove him in on one side and—bam!—out comes a dead Socrates on the other. But you don’t need people, you don’t need Socrates, you could even work with letters.

Here I have represented an “anagrammaton,” a machine that calculates anagrams (Figure 1). As you know, an anagram is something that replaces one letter with another.

To make this example as simple as possible, I’d like to propose an agrammaton the alphabet of which consists of only four letters (**A**, **B**, **C**, **D**) and which in accordance with the table in Figure 1 makes a **B** out of an **A**, a **C** out of a **B**, a **D** out of a **C**, and finally an **A** out of a **D**. When I was just a kid and sent loveletters to my girlfriend, I of course agreed with her on an anagram so that our parents couldn’t read what we wrote. But of course such anagrams are very easy to solve. For example, how many anagrams can one construct altogether with 4 letters? As you know, that’s simply the number of permutations of the letters **A**, **B**, **C** and **D**. Which is 4 times 3 times 2, therefore 4!, which results in 24 anagrams (Figure 2).

Here I have exactly 24 anagrams at my disposal, and if you want to make an experiment to find out which of these is our own anagrammaton, then you need only four trials. You give it **A**, **B** comes out; you give it **B**, **C** comes out; give it **C**, **D** comes out; and finally **A** results from giving it **D**. So you’ve solved the problem. Trivial machines are, therefore, as formulated in Proposition 01, synthetically determined (we have in fact just built one); independent of the past (we could put ours on the shelf for years and years); analytically determinable (we just did that); and, therefore, predictable.

Now you understand the great love affair of western culture for trivial machines. I could give you example after example of trivial machines. When you buy an automobile, you of course demand of the seller a trivializations-

01 02 03 04 05 06 07 08 09 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
 A A A A A A B B B B B B C C C C C D D D D D
 B B C C D D A A C C D D A A B B D D A A B B C C
 D D B D B C C D A D A C B D A D A B B C A C A B
 D C D B C B D C D A C A D B D A B A C B C A B A

The 24 anagrams that can be formed with exactly 4 letters

FIGURE 2.

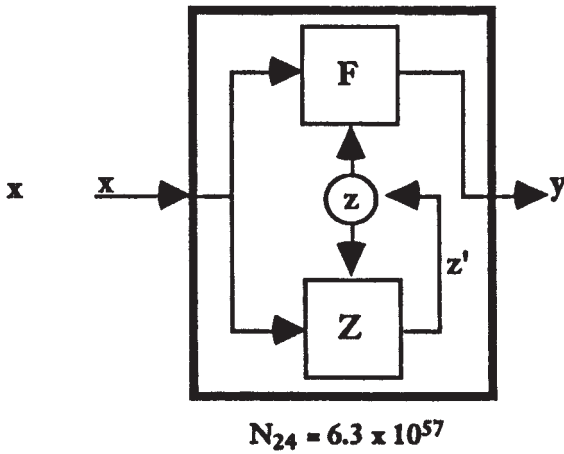


FIGURE 3.

document that says that this automobile will remain a trivial machine for the next 10,000 or 100,000 kilometers or the next five years. And if the automobile suddenly proves to be unreliable, you get a trivializateur, who puts the machine back in order. Our infatuation with trivial machines goes so far that we send our children, who are usually very unpredictable and completely surprising fellows, to trivialization institutes, so that the child, when one asks "how much is 2 times 3" doesn't say "green" or "that's how old I am" but rather says, bravely, "six." And so the child becomes a reliable member of our society.

02. Non-trivial machines: (i) synthetically determined; (ii) dependent on the past; (iii) analytically determinable; (iv) unpredictable.

Now I come to the non-trivial machines. Non-trivial machines have "inner" states (Figure 3).

In each operation, this inner state changes, so that when the next operation takes place, the previous operation is not repeated, but rather another operation can take place. One could ask, how many such non-trivial machines one could construct if, as in our case, one has the possibility of 24 different states. The number of such possible machines is $N_{24} = 6.3 \times 10^{57}$. That is a number with 57 zeros tacked on. And you can already see that some difficulties arise when you want to explore this machine analytically. If you pose a question to this machine every microsecond and have a very fast computer that can tell you in one microsecond what kind of a machine it is, yes or no, then all the time since the world began is not enough to analyze this machine. Therefore my next proposition runs:

03. Numbers: Let n be the number of input and output symbols, then the number N_T of possible trivial machines, and the number N_{NT} of non-trivial machines is: $N_T(n) = n^n$, $N_{NT}(n) = n^{nz}$, where z signifies the number of internal states of the NT machine, but z cannot be greater than the number of possible trivial machines, so that $z_{maz} = n^n$, $N_{NT}(n) = n^{nn^n}$.

For a trivial anagrammaton ($z = 1$) with 4 letters ($n = 4$) the result is $N_T(4) = 4^4 = 2^{2^4} = 2^8 = 256$.

For a non-trivial anagrammaton (which calculated different anagrams according to prescribed rules): $N_{NT}(4) = 4^{4^{4^4}} = 2^{2^{2 \cdot 2 \cdot 256}} = 2^{2048} = \text{approx. } 10^{620}$.

W. Ross Ashby, who worked with me at the Biological Computer Laboratory, built a little machine with 4 outputs, 4 inputs, and 4 inner states, and gave this machine to the graduate students, who wanted to work with him. He told them, they were to figure out for him how this machine worked, he'd be back in the morning. Now, I was a night person, I've always gotten to the lab only around noon and then gone home around 1, 2, or 3 in the morning. So I saw these poor creatures sitting and working and writing up tables and I told them: "Forget it! You can't figure it out!"—"No, no, I've almost got it already!" At six A.M. the next morning they were still sitting there, pale and green. The next day Ross Ashby said to them: "Forget it! I'll tell you how many possibilities you have: 10^{126} ." So then they relaxed.

Just imagine! Here we're concerned with only 4 letters, for input and output symbols and with inner states totaling only 24 possibilities. The complexity of this system is so enormous that it is impossible to find out how this machine works. And yet, although our brain employs over 10^{10} neurons, the representatives of "artificial intelligence" have the nerve to say that they're about to discover how the brain works. They say, "I've worked on a machine that works like the brain." "Oh, congratulations—and by the way, just how *does* the brain work?" No one knows that. But then one can't even make the comparison. One can only say that the *machine* works thus and thus, but one can't say how the *brain* works, because nobody knows. But perhaps one doesn't need to know how the brain works. Maybe it's just, as the American saying goes, that "we're barking up the wrong tree."

For example, how is it possible that this colloquium in honor of Luhmann was announced, that one would hear various speakers talk, and, although we have no idea how the brain works, that we all arrived here promptly at 9 o'clock. And what do we see? Everyone is here, everyone is listening, one of them makes noises with his mouth, some are taking notes, etc. Indeed, how is that possible? What's going on here?

For this I'd like now to take the next step. As I hope to show you, all of this can only happen because these systems operate recursively!

Second Chapter: Recursors

In order to develop the following thoughts as clearly as possible, I will increase their complexity stepwise, so that you can follow, step by step, what it's all about.

Dimensionality 1 (Operationally Open)

I'll begin with systems of dimensionality 1. Why “dimensionality 1?” Because here signals are linear and flow in one direction only. One could represent this situation in its brutal simplicity by a directed line segment (Figure 4) in which all operations that transform x into y are comprised by the one single point “0”.

Since it is my intention here to talk later on about compositions of at least two systems, I'll present you now with the two systems **D** and **S**, which should help me out in the following exposition (Figure 5).

D operates on the variable x and produces y , which is expressed by the function $y = D(x)$. The same holds, *mutatis mutandis*, for machine **S**.

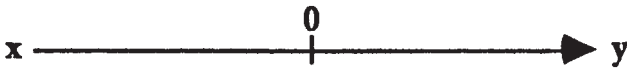


FIGURE 4.

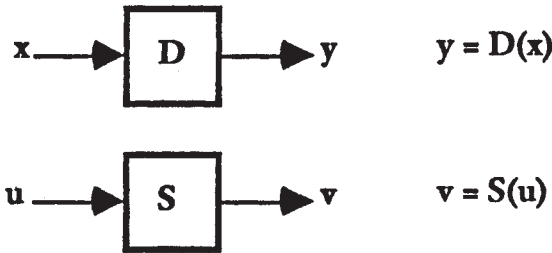


FIGURE 5.

The letters have an historical foundation, for in the development of recursive machines or even non-trivial machines one distinguished between two functions: between the state function, the **S**-function, and driving function, the **D**-function. Therefore they're called **D** and **S**, but you don't need to worry about the **D** and the **S**. You only have to know that it's about two machines, one of which operates on **x** and produces **y** while the other operates on **u** and produces **v**.

Parameterization

Now we add to these machines another, external control value, so that we can vary the operation of these machines (Figure 6). The control functions **u** and **x** that have been introduced into the machines from above are to change the operations of these machines. Regarded in another way, these parameters allow us to control the non-triviality of these machines. If these machines have the menu of our 24 anagrams at their disposal, one could switch from anagram to anagram via parametric inputs, just as one switches from channel to channel watching television. That can be expressed in two ways using an algebraic formalism. In the first, the parameter can be indicated by a subscript that modifies the function: $y = D_u(x)$, $v = S_x(u)$, in the other, it can be declared as a full-grown variable: $y = D(x,u)$, $v = S(u,x)$.

Dimensionality 2 (Operationally Closed: the Fundamental Equations of Non-linear Dynamics)

Now comes a decisive step, for I'll transform the systems, up till now of dimensionality 1, into systems of dimensionality 2 through an operational closure in which each output becomes the next input just as soon as it is produced (Figure 7).

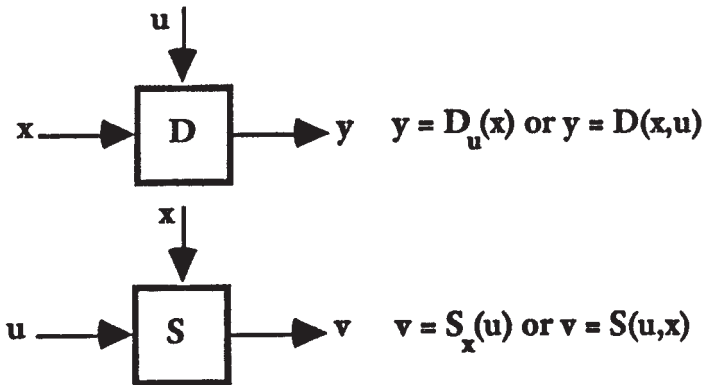


FIGURE 6.

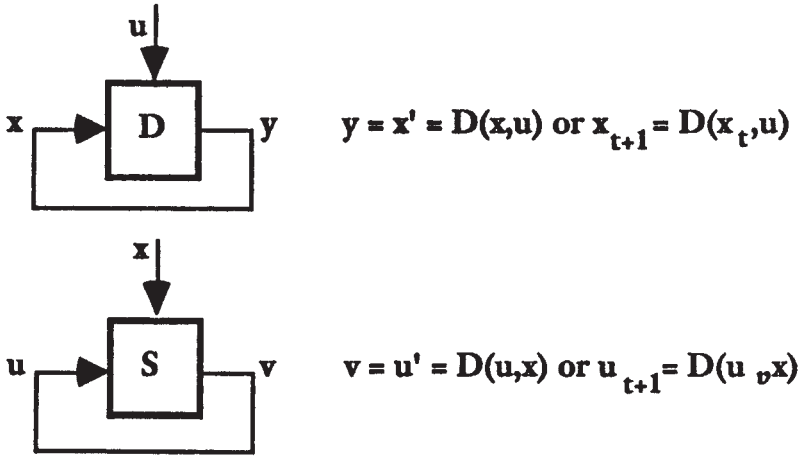


FIGURE 7.

I let y , the output of the D machine, become in turn the input. I do the same thing with the S machine. This step transforms operational linearity into operational circularity, a situation that can be represented only in a plane, therefore in a 2-dimensional manifold. Once again, this can be expressed in algebraic formalism in two ways: first, in that one makes the next input, x' , the result of the current operation, $x' = y$, whereby the marked quantity is to follow the unmarked one:

$$x' = D(x, u), \text{ and } u' = S(u, x)$$

whereby the recursiveness of these expression can be recognized in that the variables x, u appear as functions of themselves. One gets a "physicalization" of this situation, expressing the passage of time, by introducing the parameter "time" in the form of incremental units: t now, $t + 1$ a single incremental unit later:

$$x_{t+1} = D(x_t, u), \text{ and } u_{t+1} = S(u_t, x)$$

Those of you who are occupied with chaos theory and with recursive functions will recognize at once that these are the fundamental equations of recursive function theory. Those are the conceptual mechanisms with which chaos research is conducted; it is always the same equations over and over again. And they give rise to completely astonishing, unforeseen operational properties. Viewed historically, even early on one noticed a convergence to some stable values. An example: if you recursively take the square root of any random initial value (most calculators have a square root button), then you will very soon arrive at the stable value 1.0000. . . . No wonder, for the root of 1 is 1. The mathematicians at the turn of the century called these values the "Eigen values" of the corresponding functions. To the operation

of taking roots belong the Eigen values 1 and also 0, since any root of 0 is 0. The essential difference between these two Eigen values is that for every deviation from 1, recursion leads the system back to 1, while at the least deviation from 0 the system leaves null and wanders to the stable Eigen value “one.”

About 20 years ago there was an explosion of renewed interest in these recursive operations, as one discovered that many functions develop not only stable *values* but also a stable *dynamic*. One called these stabilities “attractors,” apparently a leftover from a teleological way of thinking. Since one can let some systems march through the most diverse Eigen behaviors by making simple changes in the parameters, one soon stumbled onto a most interesting behavior that is launched by certain parametric values: the system rolls through a sequence of values without ever repeating one, and even if one believes one has taken one of these values as the initial value, the sequence of values cannot be reproduced: the system is chaotic.

Let me make just a couple of more remarks about stable Eigen behaviors.

Consider next the fascinating process that recursively sifts only discrete values out of a continuum of endless possibilities. Recall the operation of taking roots, which lets one and only one number, namely “1,” emerge from the endless domain of the real numbers. Can that serve as a metaphor for the recursiveness of the natural process, sometimes also called “evolution,” in which discrete entities are sifted out of the infinite abundance of possibilities, such as a fly, an elephant, even a Luhmann? I say yes,” and hope to contribute additional building blocks to the foundation of my assertion.

But consider also that although one can indeed make the inference from given operations to their Eigen behaviors, one cannot make the converse deduction from a stable behavior, an Eigen behavior, to the corresponding generative operations. For example, “one” is the Eigen value of infinitely many different operations. Therefore, the inference from the recursive Eigen value “1” to the square root operation as the generator is not valid, because the fourth, the tenth, the hundredth root, recursively applied, yield the same Eigen value “1.” Can that serve as a metaphor for the recursiveness of the natural process, sometimes called the “laws of nature,” of which there could be infinitely many versions that would explain a Milky Way, a planetary system, indeed, even a Luhmann? I say “yes” and turn for support to Wittgenstein’s *Tractatus*, Point 5.1361: “The belief in the causal nexus is *the* superstition.”

This result, that there emerge Eigen values, is the only thing we can rely on. For then an opaque machine begins to behave in a predictable way, for as soon as it has run into an Eigen state, I can of course tell you, for example, if this Eigen state is a period, what the next value in the period is. Through this recursive closure and only through this recursive closure do stabilities arise that could never be discovered through input/output analysis. What is fascinating is that while one can observe these stabilities it is in principle

impossible to find out what generates these stabilities. One cannot analytically determine how this system operates, although we see that it does operate in a way that permits us to make predictions.

Third Chapter: Compositions

I have up till now spoken of systems as entities, spoken about their behavior, their synthesis, analysis, and taxonomy. But here I am in the company of scholars of sociology, i.e., the science of the “*socius*,” of the companion and comrade, of the “*secundus*,” the follower, the second. So I must concern myself with at least two systems, with their behavior, their synthesis and analysis. Indeed a society usually consists of more than two members, but if the process of integration, the “composition” of two systems has been established, one can use stepwise recursion to apply the established composition rule to an arbitrary number of new arrivals.

How does such a composition come about?

Here, I believe, is perhaps the essential step in my exposition, for through the composition of two systems of dimensionality 2, the recursors, there emerge systems that are irreducibly of dimensionality 3.

But how is one to proceed?

Dimensionality 3 (Calculus of Recursive Functors)

The systems in Figure 8 should help me out here. I’ll go back to the two machines, the recursors **D** and **S** from Figure 7.

In step one (orientation) I rotate recursor **S** 90°, so that the variables and parameters in **D** and **S** are aligned with one another; in step two (composition) I push the two together, so that out of the two separate systems **D** and **S** a new machine now arises, a **DS**-composition. This new machine is distinguished by its double closure, first a closure on **u**, that previously, as a parameter, controlled **D**, and then the closure on **x**, that previously, as a parameter, controlled **S**. So now both systems control one another reciprocally; the operational functions of the one system become functions of the other: two recursive functors.

Extensions of the Second Order

0.5 Functors: functions of functions (functions of the second order)

From your middle school years you can surely recall the differential and integral calculus. One wrote $\mathbf{dy/dx}$ and spoke of the “derivative of **y** with respect to **x**,” whereby **y** is a function of **x**: $\mathbf{y} = \mathbf{f}(\mathbf{x})$. That is, the derivative, or differential operator **Di**, as I’d like to call it, is a functor, for it operates

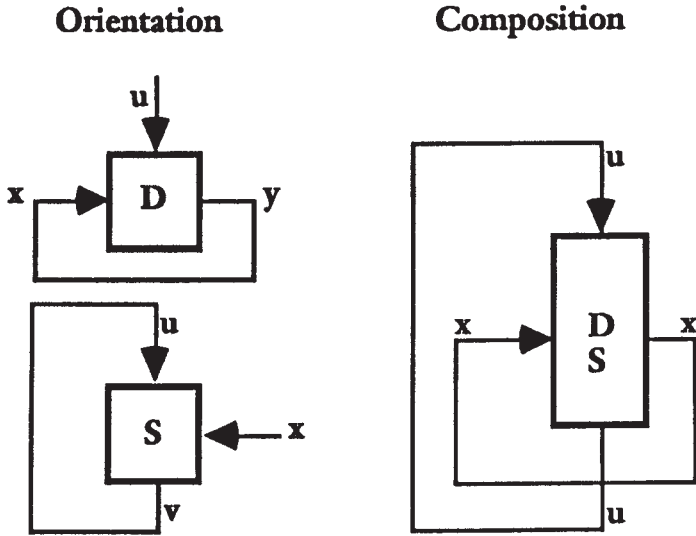


FIGURE 8.

on a function, let's say $y = x^2$, and produces a function: $\mathbf{Di}[x^2] = 2x$, or in Menger's elegant notation: $\mathbf{Di}[()^2] = 2$. Does \mathbf{Di} have Eigen functions? Yes indeed: the exponential function $y = e^x$, in Menger's notation: $\mathbf{Di}[\mathbf{exp}] = \mathbf{exp}$, and on account of the extraordinary relationship of the exponential function to the trigonometric functions \mathbf{sin} and \mathbf{cosin} : $\mathbf{Di}^4[\mathbf{sin}] = \mathbf{sin}$, $\mathbf{Di}^4[\mathbf{cos}] = \mathbf{cos}$, i.e., \mathbf{sin} and \mathbf{cosin} are the Eigen functions of the differential operator iterated fourfold.

One doesn't have to restrict oneself to mathematical expressions. Menger developed these ideas for logical functions (1962), a generalization that is significant here. For example, the algebraic expression:

$$\begin{array}{c}
 \downarrow \\
 \mathbf{S} = \mathbf{S}(\mathbf{D}) = \mathbf{S}(\mathbf{D}(\mathbf{S})) \\
 \mathbf{D} = \mathbf{D}(\mathbf{S}) = \mathbf{D}(\mathbf{S}(\mathbf{D})) \\
 \uparrow
 \end{array}$$

of the composition of the two systems \mathbf{D} and \mathbf{S} in Figure 8 makes the recursion of the two functors \mathbf{D} , \mathbf{S} clearly visible.

06. Compositions (the properties of the composition are not the properties of the components)

Viewed historically, attention to qualitative changes that arise in the transition from aggregate to system was guided by an unfortunate formulation of this transition that was given by "generalists," "holists," "environmental-

ists,” etc.: “the whole is greater than the sum of its parts.” As one of my colleagues once remarked: “can’t the numskulls even add?”

But if a measure function **M** is introduced, then the holistic sentiment can be made precise: “The measure of the sum of the parts is not the sum of the measures of the parts”: $\mathbf{M}\Sigma(\mathbf{T}_i) \neq \Sigma\mathbf{M}(\mathbf{T}_i)$, ($i = 1, 2, 3, 4 \dots n$). If the measure function is super-additive, then indeed the holistic motto is justified. Let us take two parts (**a**, **b**) and for our measure function squaring (\square). Then in fact $(\mathbf{a} + \mathbf{b})^2$ is greater than $\mathbf{a}^2 + \mathbf{b}^2$, for $\mathbf{a}^2 + \mathbf{b}^2 + 2\mathbf{ab}$ is greater than $\mathbf{a}^2 + \mathbf{b}^2$, and indeed by exactly the systemic reciprocity part $\mathbf{ab} + \mathbf{ba}$, which, by symmetry (commutativity $\mathbf{ab} = \mathbf{ba}$): $\mathbf{ab} + \mathbf{ba} = 2\mathbf{ab}$.

A first step in the generalization of the measure function permits us to establish the rules of the game of an algebra of composition, in which one, as previously, regards the distributive law only as a special case vis-à-vis operators. If **K** is some composition (addition, multiplication, logical implication, etc.), then, just as previously: $\mathbf{Op}[\mathbf{K}(\mathbf{f}, \mathbf{g})] \neq \mathbf{K}[\mathbf{Op}(\mathbf{f}), \mathbf{Op}(\mathbf{g})]$.

That is to say, the result of an operation **Op** on a system constructed via the **K**-composition is not equivalent to a system constructed via the **K**-composition of the results of the operator **Op**.

This proposition plays an important role for the autopoieticists, who indeed always insist that the properties of the autopoietic system cannot be expressed by the properties of its components.

Now just two cases worth mentioning (a restriction and an extension), which together allow the interchange of operations and compositions.

- (i) Homogeneous composition: let **K** be the composition rule, then $\mathbf{Op}[\mathbf{K}(\mathbf{f}, \mathbf{g})] = \mathbf{K}[\mathbf{Op}(\mathbf{f}), \mathbf{Op}(\mathbf{g})]$;
- (ii) Superposition: Let **K** and **C** be composition rules, then $\mathbf{Op}[\mathbf{K}(\mathbf{f}, \mathbf{g})] = \mathbf{C}[\mathbf{Op}(\mathbf{f}), \mathbf{Op}(\mathbf{g})]$.

This formulation moved the inventors of information theory to follow the example of Boltzmann and choose the logarithmic function for the entropy **H** (here **Op**) of a signal source. Since when two sources with signal repertoires $\mathbf{n}_1, \mathbf{n}_2$ are composed, the new repertoire is $\mathbf{n}_1 \times \mathbf{n}_2$, the new entropy is simply the sum of the former two: $\mathbf{H}(\mathbf{n}_1 \times \mathbf{n}_2) = \mathbf{H}(\mathbf{n}_1) + \mathbf{H}(\mathbf{n}_2)$, for $\log(\mathbf{a} \cdot \mathbf{b}) = \log(\mathbf{a}) + \log(\mathbf{b})$.

If you consider the “composition” in Figure 8 more closely, you will see that it is in principle impossible to arrange the **x** and the **u** loops on the paper in such a way that they don’t intersect one another. One must raise either the **x** or the **u** off the paper into the “third dimension” in order to add the two recursions to the system in such a way that they are independent of one another. This can be made even clearer if one dispenses with drawing the external lines, in that one rolls the **DS**-system into a cylinder around the **u**-axis, so that the **x**-output and **y**-input edges are merged.

One can also get rid of the outer **u**-loop in that one bends the cylinder into a ring and melds the upper and lower circular ends: this makes **u**-out

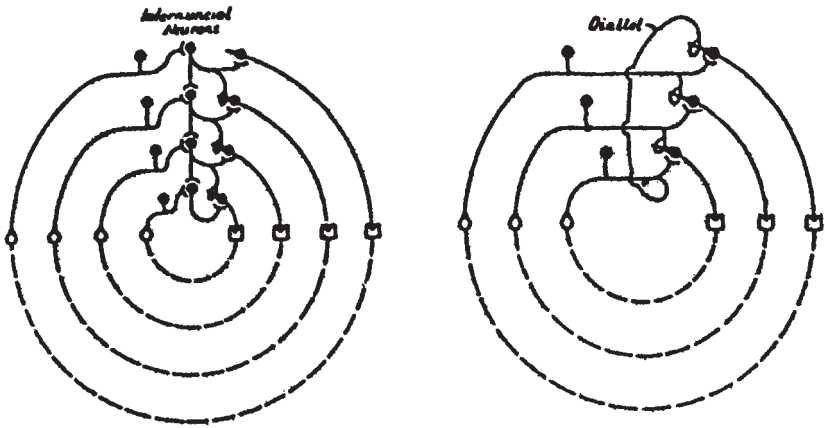


FIGURE 9.

into **u-in**. This ring or torus is the topological representative of a doubly closed system.

If you'd like pictures, you can find them already very early in Warren McCulloch's article "A Hierarchy of Values Determined by the Topology of Nervous Nets":

07. Warren S. McCulloch (1945): "A Hierarchy of Values Determined by the Topology of Nervous Nets" (Figure 9).

His argument is as follows: In his Figure 3 (Figure 9, left), he shows the recursion of neural activity whose internal components are indicated by the unbroken arcs and whose external components are indicated by the broken ones: McCulloch's thesis of the closure of the neural pathways via the environment. In this circuit the organization is hierarchical, for the presently external senso-motoric loops (dromes) can inhibit the inner loops. Therefore this network cannot calculate the "circularities in preference," the "value anomaly" that I spoke of. In his Figure 4 (Figure 9, right), he introduces the diallels ("crossovers") that from the lower circle can inhibit the upper: twofold closure.

A second reference to the value of toroids for representing doubly closed processes will be found in Proposition 8:

08. Double closure of the senso-motoric and inner-secretoric-neuronal circuits. N = neural bundle; syn = synapse; NP = neurohypophysis; MS = motorium; SS = sensorium (Figure 10).

Here you see sketched (Figure 10a) both of the orthogonally operating circuits: on one side the neural signal flow from the sense organs (**SS**) via the nerve bundles (separated by synapses) to the motorium and from there through the environment and back to the sensorium (**SS**); on the other side

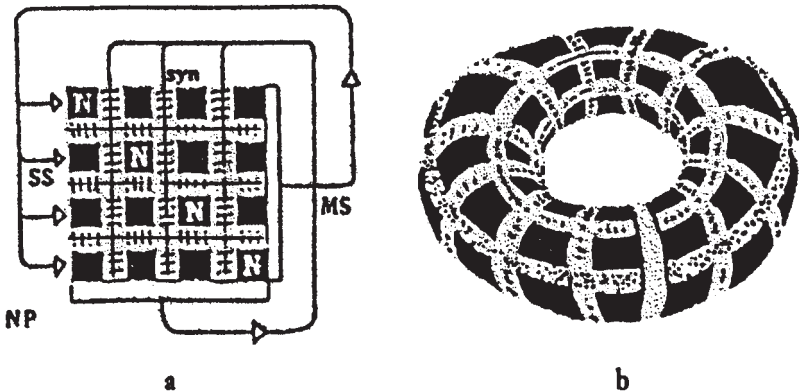


FIGURE 10.

the steroids that are poured into the synapses by the neurohypophysis, which recursively regulate the neural transition functions. Once again, we obtain the torus when we wrap the square scheme around both the horizontal and the vertical axes (Figure 10b).

09. The closure theorem: "In every operationally closed system there arise Eigen behaviors."

Among the many variants and paraphrases of this astonishing theorem I've picked Francisco Varela and Joseph Goguen's version, for I believe I see an affinity here with the sociological vocabulary. The word "behavior," as well as "conduct," "action," etc., does imply the recognizability of regularities, of "invariants" in the temporal course of the action. Here, among sociologists, one is probably not as interested in whether the cosin or the sin appears as the Eigen behavior of the system, but rather whether in a cultural domain a meeting between two members of this cultural domain is celebrated by a handshake or by bowing.

One could even go further and be on the lookout for the emergence of invariants that arise when air is blown in a certain way through the vocal cords, whose vibrations then elicit hisses and grunt with which the meeting of two members of a cultural domain is celebrated and in the southern regions of this geographic area are heard as "Hi, y'all" and in the northern regions as "Hello there."

In all that I have said up until now, I have tried to make it obvious that these invariants, these "Eigen behaviors" arise through the recursively reciprocal effect of the participants in such an established social domain. Therefore, I'd like to turn back to the original question that was put to me: "How recursive is communication?" and also to my proposal:

00. Communication is recursion

With the vocabulary developed here I can extend and sharpen this first version with a few words:

10. Communication is the Eigen behavior of a recursively operating system that is doubly closed onto itself.

The essential thing about the topology of a double closure is that it not only avoids the pseudo-solution of hierarchy, to which one always defers the responsibility for judgment in order to avoid one's own, but also that, through the hetarchical organization that comes with it, the fascinating possibility exists of allowing operators to become operands and operands to become operators. This is just exactly what we've always wanted to understand but which has nevertheless been made impossible for us up till now by the structure of a one dimensional logic. But through the interchangeability of functors standing in reciprocal relationships to one another, our freedom of action is returned to us and with it also our responsibility.

With that I have arrived at my conclusion, which I owe to Wilhelm Busch:

11. Wilhelm Busch's Desideratum:

“Twice two is four” is clearly true,
Too bad it's cheap and flighty;
For I would rather that I knew
About what's deep and mighty.

Whether I've succeeded at that, I don't know, but I do thank you many times for having been so friendly as to have the patience to listen to me. And as my last word, I'd like once more to congratulate our birthday child Niklas Luhmann.

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